

EKOS Linear 1 Pass Focus Phase 2

Introduction

Linear 1 Pass Focus Phase 2 builds on the work done for Linear 1 Pass to optimise the routine and add new functionality.

Summary of Changes

Item	What's this?	Comments
Curve Fitting	Currently Levensburg-Marquardt curve fitting is applied to Hyperbola and Parabola. This has been optimised.	Optimisation of curve fitting: <ul style="list-style-type: none"> - Geodesic acceleration. Second order differential terms have been added to the LM solver which makes curve fitting faster and more accurate. Thanks to work by Sophie Taylor. - Other LM parameters have been tweaked to optimise the fitting process. - Outlier detection and removal. - Robust statistics have been applied. Thanks to Sophie Taylor for the stats library.
Fitting Measure	Currently HFR. FWHM, Num Stars and HFR Adj have been added.	FWHM will be a v-curve similar to HFR. Initially, a gaussian curve will be used. Num stars is an inverted v-curve. HFR Adj is similar to HFR. Brighter stars are bigger than fainter ones. The adjustment aims to adjust the measured HFR to compensate for brightness so that the comparison for in focus (brighter) stars against out-of-focus (fainter) stars is more valid.
Focus Units	Currently Pixels. Arc-seconds has been added	Relevant to both HFR and FWHM.
CFZ	Critical Focus Zone (CFZ) has been added. Several algorithms are available.	The CFZ is defaulted from the selected optical train. Parameters can be adjusted for what-if type analysis. The CFZ can be displayed on the v-curve.
Focus Offset Utility	A utility to automatically build focus offsets.	Based on the filter setup, Build Offsets will pull out Lock Filters and Dependent Filters and then run Autofocus a user-defined number of times on each filter, average the focus points, then calculate and persist the offsets.
Flats	Take flats at same focus point as lights.	This utility has been fixed. In addition, the last used flat position can now be adapted as per AF Start.
Focus Adviser	Utility to advise on focus parameter setup.	This is an experimental feature and will rely heavily on user feedback to develop going forward. The concept is to analyse a focus run and suggest parameter improvements. Targeted at novice users.
SEP Params	SEP parameters suggestions for focus.	
Adaptive Focus	Adaptive focus will adapt the focuser position between Autofocus runs, aiming to keep optimum focus for each sub-frame. Adaptations for Temperature and Altitude are supported.	When switched on, Adaptive Focus will check between sub-frames whether conditions dictate a focus adjustment. If so, this will be performed. This is an experimental feature and is implemented in its most basic form.
AF Start	The starting position for an AF run will be filter and Adaptive Focus adjusted.	Autofocus works best if it is started near to optimum focus. Optimum focus is filter, temperature and possibly altitude dependent.
Walk	How the inward sweep of the AF process performs. Currently, a "do your best to find optimum focus" walk is performed.	The following additional walks have been added: <ul style="list-style-type: none"> - Exact number of steps. Good for minimising unnecessary steps. - Adjust step size to take smaller steps nearer to optimum focus. This is experimental.
Overscan	Overscan was originally implemented in the Linear algorithm and then reused by Linear 1 Pass	If set, overscan will apply to all outward focus movements.

Curve Fitting

Currently Levensburg-Marquardt curve fitting is applied to Hyperbola and Parabola. This has been optimised:

- Geodesic acceleration. Second order differential terms have been added to the LM solver which makes curve fitting faster and more accurate.
- Optimisation of other solver parameters: Solver, Scale and Trust Region have been evaluated with a view to optimising curve fitting. Some time has been spent on convergence thresholds.
- Refine Curve Fit. The purpose of this is to provide a more robust exclusion of outlier points to make curve fitting more accurate in the presence of outliers. Check the “Refine Curve Fit” checkbox to activate. After the pass has completed (in the below example after 11 points), Ekos does a simple 1-pass sigma clip (at 2 sigma) of outlier points. For less than 11 points upto 2 points could be removed; greater than 11 points upto 3 points could be removed. Curve fitting is re-applied on the remaining data points. If the result has a better R^2 than the original this new solution is kept. In this example, points 1, 5 and 7 were removed.

The screenshot shows the Ekos - Simulators Profile interface. The 'Refine Curve Fit' checkbox is checked. The V-Curve graph shows the HFR (pix) vs. Focus Position (pix) with 11 points and a hyperbola fit. The HFR is 0.96, FWHM is 0.00, Stars is 11, and Iteration is 12. The R² value is 0.99.

2023-02-18T16:15:00 Focus procedure completed after 12 iterations.
2023-02-18T16:15:00 Detection complete.
2023-02-18T16:15:00 Detecting sources...
2023-02-18T16:15:00 Image received.
2023-02-18T16:14:59 Capturing image...

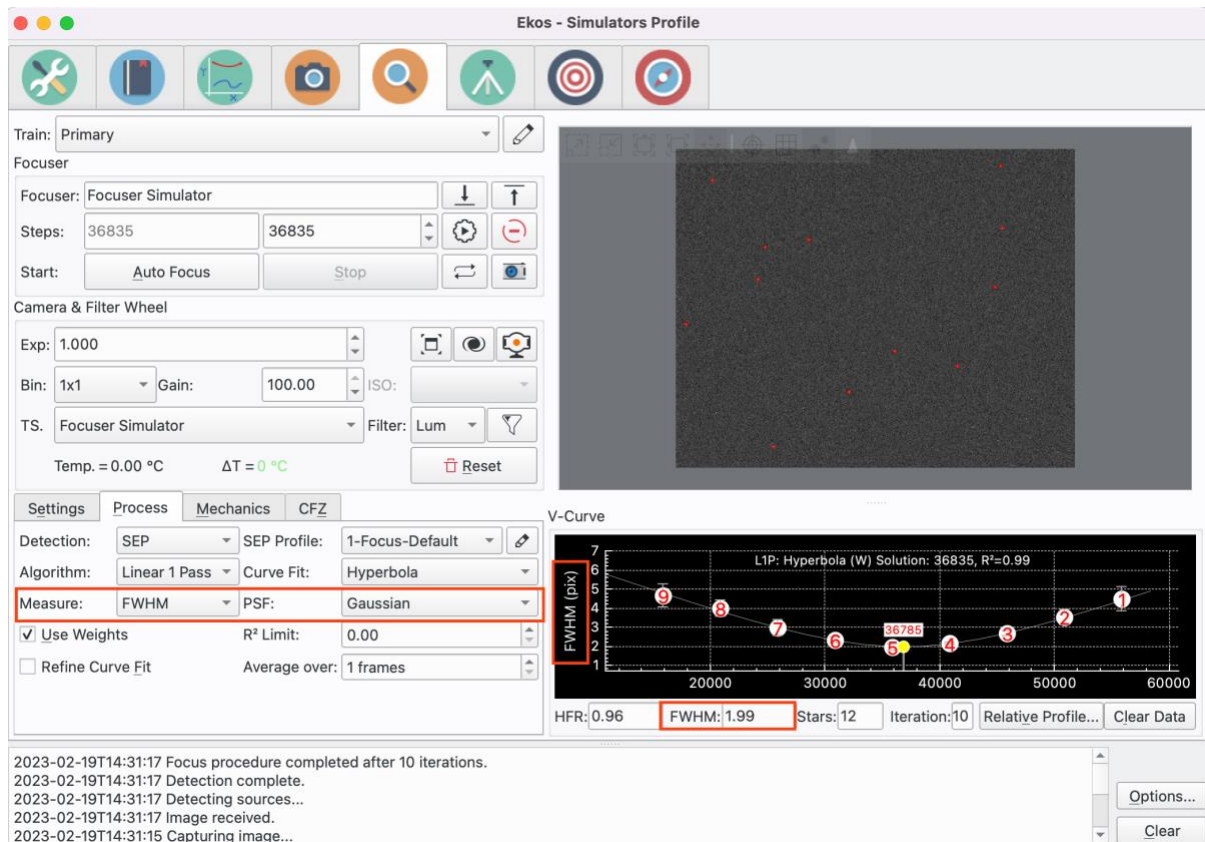
Fitting Measure

In addition to HFR, FWHM and Number Stars are now available along with a modified HFR. These are only available with the Linear 1 Pass algorithm.

FWHM

Select “FWHM” as the Measure in the Process Tab. Currently just “Gaussian” curve fitting is supported but this may be extended in future to Moffat stars. The graph axis changes to “FWHM” and the measure used to calculate the best focus position is FWHM. The HFR will

still be calculated and the average value of HFR and FWHM for each frame is displayed under the curve.



Curve fitting is used to fit a PSF to each star in the frame. This is a CPU intensive process so may not be suitable for lower powered machines. For this reason FWHM values are only calculated if the Measure is set to FWHM. The algorithm is quite generic so could be optimised in future if required, but by starting with a generic algorithm it should be possible to reuse it in future for other purposes.

In this implementation the output of HFR processing is used to seed the curve fitting process to make it more efficient, e.g. star centroiding. In future, this processing may be incorporated into the Fits viewer to make it a more widely available alternative to HFR.

See Appendix 1 – FWHM on page 17 for more details.

Number Stars

Select “# Stars” in the Process Tab. The measure used is the total number of stars detected in each frame. As the focuser moves closer to focus so the number of stars increases. The shape of the curve is an inverted V-curve and the point of optimal focus corresponds to the curve maximum.

[Insert screenshots]

HFR Adjusted

Select “HFR Adj” in the Process Tab. The graph axis changes to “HFR Adj” and the measure used to calculate the best focus position is an intensity adjusted HFR. The basic idea is that brighter stars appear bigger than fainter stars. So, when averaging a frame of stars of varying magnitudes, it would be more appropriate to intensity adjust the HFRs before averaging and then comparing various focus positions. This is described in more detail in Appendix 2 – Star Magnitude Compensation on page 23.

The screenshot shows the Ekos - Simulators Profile software interface. The main window displays a simulated star field with numerical values for each star, such as 0.97, 0.90, 0.98, 0.92, 0.94, 0.96, 0.99, 1.04, 1.01, and 1.02. The interface includes a 'Focuser' section with 'Focuser Simulator' selected, 'Steps' set to 36805, and 'Start' set to 'Auto Focus'. The 'Camera & Filter Wheel' section shows 'Exp: 1.000', 'Bin: 1x1', 'Gain: 100.00', 'ISO: [blank]', 'TS: Focuser Simulator', and 'Filter: Lum'. The 'Settings' tab is active, showing 'Detection: SEP', 'SEP Profile: 1-Focus-Default', 'Algorithm: Linear 1 Pass', 'Curve Fit: Hyperbola', 'Measure: HFR Adj' (highlighted with a red box), 'PSF: Gaussian', 'Use Weights' checked, 'R² Limit: 0.00', and 'Average over: 1 frames'. The 'V-Curve' graph shows 'HFR Adj (pix)' on the y-axis and 'Iteration' on the x-axis, with a hyperbolic fit line and data points numbered 1 through 9. The graph title is 'L1P: Hyperbola (W) Solution: 36805, R²=0.99'. Below the graph, the 'HFR: 0.99' is displayed. The bottom status bar shows the focus procedure completed after 10 iterations.

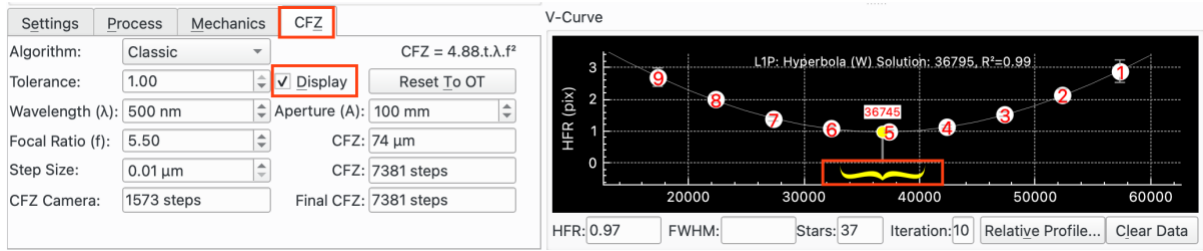
Focus Units

An ability to view HFR and FWHM information in arc-seconds (as well as pixels) has been added. Select “Arc Seconds” or “Pixels” in the Settings tab. The conversion from pixels to arc-seconds is derived from the optical train being used for focus.

The screenshot shows the Ekos - Simulators Profile software interface with the 'Settings' tab active. The 'Process' section is expanded, showing 'Auto Select Star' checked, 'Dark Frame' unchecked, 'Sub Frame' unchecked, 'Full Field' checked, 'Suspend Guiding' checked, 'Adaptive Focus' unchecked, and 'Adapt Start Pos' unchecked. The 'Mechanics' section shows 'Box: 256 px', 'Annulus: 0.0 %', 'Guide Settle: 1.00 s', 'Display Units: Arc Seconds' (highlighted with a red box), 'Min Move: 1 ticks', and 'Max Total Move: 1000 ticks'. The 'CFZ' section is empty. The 'V-Curve' graph shows 'FWHM (")' on the y-axis and 'Iteration' on the x-axis, with a hyperbolic fit line and data points numbered 1 through 9. The graph title is 'L1P: Hyperbola (W) Solution: 36929, R²=0.99'. Below the graph, the 'FWHM: 4.08' is displayed. The bottom status bar shows the focus procedure completed after 10 iterations.

Critical Focus Zone

Some calculations for CFZ have been added as a separate CFZ tab in focus parameters.



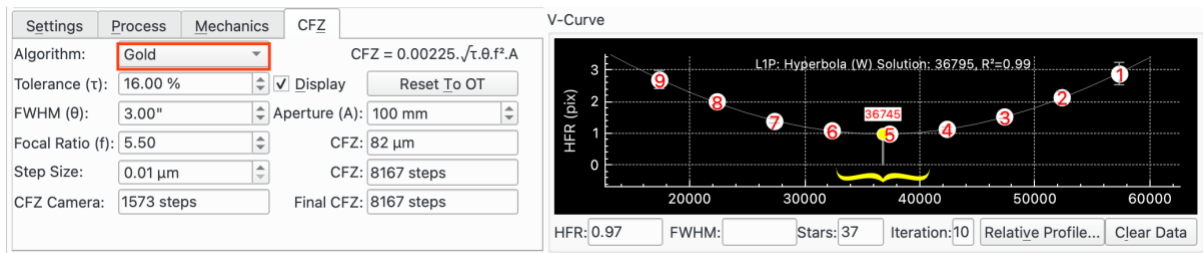
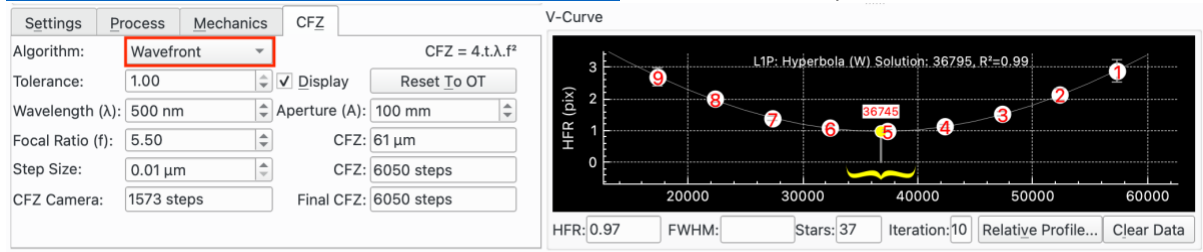
Depending on the choice of algorithm, different parameters will be required. Where appropriate, data is defaulted from the equipment profile for the active optical train, e.g. Focal Ratio, Focal Length, Aperture and pixel size of the camera. Wavelength can be set per filter in the Filter Settings popup.

Step Size in microns will need to be worked out and input by the user for their equipment. This is required to convert the calculated CFZ in microns to steps.

If the “Display” checkbox is set then the CFZ is annotated on the V-curve after a successful Autofocus as a “moustache”.

The defaulted parameters can be adjusted by the user for “what if” analysis. If the optical train is changed or the “Reset to OT” button pushed then the parameters are reset to their appropriate defaults.

In addition to the Classic algorithm a wavefront version and Gold Astro version (see <http://www.goldastro.com/goldfocus/ncfz.php> for more details) are available:



See Appendix 3 – Critical Focus Zone (CFZ) on page 30 for more details.

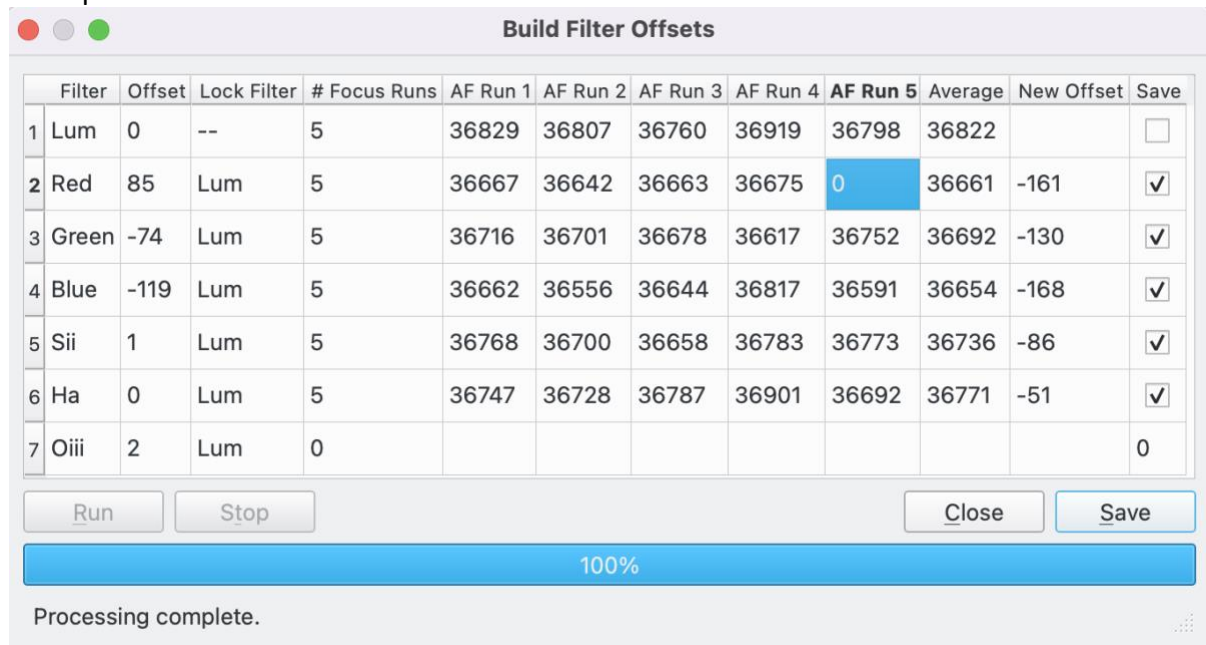
Focus Offset Utility

The purpose of this utility is to automate the process of setting up filter offsets. Filter offsets are useful:

1. To reduce time spent focusing. When a filter is selected, rather than performing an Autofocus run, the offset is applied to move the focuser to the correct position.

- Some filters (e.g. narrowband) may not focus in certain parts of the sky because of lack of stars.

Ekos already supports this concept, but the Offsets Utility helps to automate the setup of offsets. Firstly, configure the filters as you like, e.g. use Lum as your lock and other filters to be dependent filters on Lum.



	Filter	Offset	Lock Filter	# Focus Runs	AF Run 1	AF Run 2	AF Run 3	AF Run 4	AF Run 5	Average	New Offset	Save
1	Lum	0	--	5	36829	36807	36760	36919	36798	36822		<input type="checkbox"/>
2	Red	85	Lum	5	36667	36642	36663	36675	0	36661	-161	<input checked="" type="checkbox"/>
3	Green	-74	Lum	5	36716	36701	36678	36617	36752	36692	-130	<input checked="" type="checkbox"/>
4	Blue	-119	Lum	5	36662	36556	36644	36817	36591	36654	-168	<input checked="" type="checkbox"/>
5	Sii	1	Lum	5	36768	36700	36658	36783	36773	36736	-86	<input checked="" type="checkbox"/>
6	Ha	0	Lum	5	36747	36728	36787	36901	36692	36771	-51	<input checked="" type="checkbox"/>
7	Oiii	2	Lum	0								0

Run Stop Close Save

100%

Processing complete.

The utility pulls out the Lock filters and Dependant filters. The user enters the number of runs per filter (zero to ignore) and presses Run. The utility takes over the focus module to run the selected number of Autofocus runs for each filter. The exposure can be set per filter but other AF parameters should be set beforehand in the focus module.

The utility will calculate an average Autofocus position per filter and calculate the New Offset. The user can manually change the New Offset or an individual AF result and the utility will recalculate the offset(s). Set an AF result to zero to have it excluded from the average (if, for example, it is an outlier).

When done hit "Save".

The Build Filter Offsets Dialog is launched from the Filter Settings Dialog (available from Capture and Focus).

Filter Settings

Filter Wheel **Filter Simulator**

	Filter	Exposure	Offset	Auto Focus	Lock Filter	Last AF Solution	Last AF Temp	Last AF Alt	Ticks / °C	Ticks / °Alt	Wavelength
1	Lum	1	300	<input checked="" type="checkbox"/>	--	36798	0	51.5778	2	0	350
2	Red	2	-161	<input checked="" type="checkbox"/>	Lum	36573	0	51.5778	-1	1	500
3	Green	3	-130	<input checked="" type="checkbox"/>	Lum	36752	0	51.5778	0	0	500
4	Blue	1	-168	<input checked="" type="checkbox"/>	Lum	36591	0	51.5778	0	0	500
5	Sii	5	-86	<input checked="" type="checkbox"/>	Lum	36773	0	51.5778	0	0	500
6	Ha	5	-51	<input checked="" type="checkbox"/>	Lum	36692	0	51.5778	0	0	500
7	Oiii	5	2	<input checked="" type="checkbox"/>	Lum	36652	0	51.5778	0	0	500
8		0.001	0	<input type="checkbox"/>	--	36651			0	0	500

Set *relative* filter offsets in steps either directly in the table or use the Build Offsets tool. Build Offsets

Capture flats at the same focus as lights Close

Flats

Ekos has an existing utility to allow flats to be taken at the same focus as lights...

Filter Settings

Filter Wheel **Filter Simulator**

	Filter	Exposure	Offset	Auto Focus	Lock Filter	Last AF Solution	Last AF Temp	Last AF Alt	Ticks / °C	Ticks / °Alt	Wavelength
1	Lum	1	0	<input checked="" type="checkbox"/>	--	36645	0	51.5778	9	0	350
2	Red	2	0	<input checked="" type="checkbox"/>	--	36711	5	51.5778	0	0	500
3	Green	3	-130	<input checked="" type="checkbox"/>	Lum	36665	0	51.5778	0	0	500
4	Blue	1	-168	<input checked="" type="checkbox"/>	Lum	36591	0	51.5778	0	0	500
5	Sii	5	-86	<input checked="" type="checkbox"/>	Lum	36773	0	51.5778	0	0	500
6	Ha	5	-51	<input checked="" type="checkbox"/>	Lum	36692	0	51.5778	0	0	500
7	Oiii	5	2	<input checked="" type="checkbox"/>	Lum	36652	0	51.5778	0	0	500
8		0.001	0	<input type="checkbox"/>	--	36651			0	0	500

Set *relative* filter offsets in steps either directly in the table or use the Build Offsets tool. Build Offsets

Capture flats at the same focus as lights Close

This has been fixed. When “Capture flats at the same focus as lights” is checked, the “Last AF Solution” is used for the chosen filter when taking a flat.

In addition, if “AF Start” is checked and the focus algorithm is set to Linear 1 Pass, then Ekos will adapt the focus position based on temperature and altitude. See the AF Start section on page 12 for more details.

AF Overscan has been extended to this function.

Focus Advisor

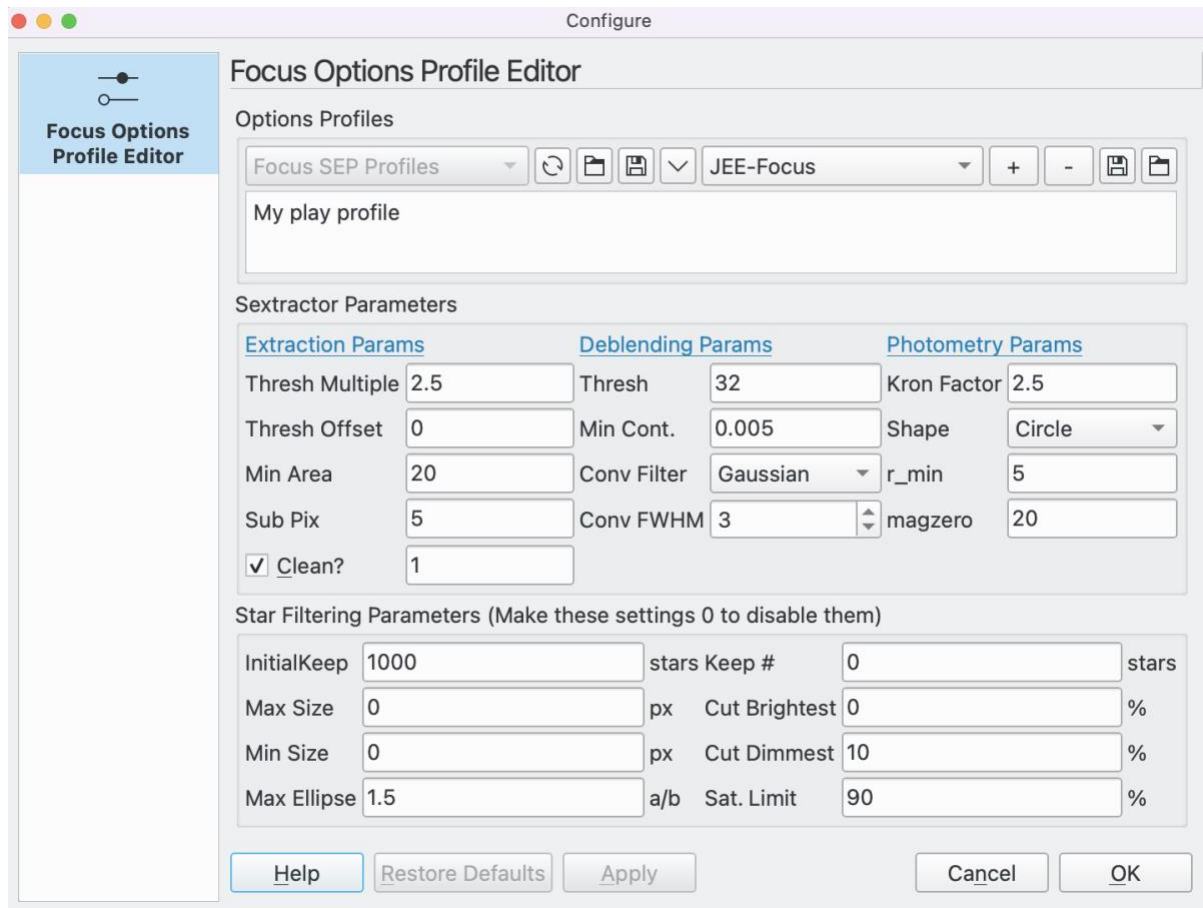
The Focus Advisor (FA) needs more work. The goal is to help new users get to a point where Autofocus will run. The FA will not find the optimum set of parameters, that will still require user experimentation.

Field	Suggestion	Comment
Camera Settings		
Exposure	1s – 5s	Analyse star field. If no saturated stars, increase. If lots, decrease
Bin		Look at image scale and suggest binning that brings scale to 1-2 "/pixel.
Gain	Unity gain	
Filter	Lum	Suggest the filter with the largest bandpass
Settings Tab		
Auto-select	On	
Full Field	On	So subframe is off
Annulus	0 – 100%	Gives the most stars
Suspend Guiding, Guide Settle, Display Units	No recommendation	Not relevant for Autofocus
Adaptive Focus	No recommendation	Not relevant for Autofocus
Process Tab		
Detection	SEP	
SEP Profile	1-Focus-Default	This generally should work fine
Algorithm	Linear 1 Pass	
Curve Fit	Hyperbola/Parabola	
Measure	HFR	
Use Weights	Checked	
R ² Limit	0.8 – 0.9	
Refine Curve Fit	Unchecked	Experimental feature
Average Over	1 frame	
Mechanics Tab		
Initial Step Size	CFZ	Set to CFZ
Out Step Multiple	4 - 5	Max HFR / Min HFR = 3 for refractor, = 2 for central obstruction telescope
Max Travel	2 * Initial Step Size * Out Step Multiple	
Driver Backlash	<Backlash value>	Set to zero if driver does not support backlash. Set to backlash value if supported
AF Overscan	2 * <Backlash value>	Set one of AF Overscan and Driver Backlash.
Focuser Settle	At least 1s	
Walk	Classic	
Capture Timeout	30s	
Motion Timeout	30s	

SEP Params

Below are thoughts on how to set SEP params for focus. Its better to setup a fresh profile so that “1-Focus-Default” remains unchanged.

Field	Suggestion	Comment
Extraction Params		
Thresh Multiple	2 – 3 (2.5 works well)	Multiplier for background noise rms as threshold for star detection. Lower to get more stars (but also more noise).
Thresh Offset	0	Alternative to Thresh Multiple.
Min area	20	Min number pixels to identify a star.
Sub pix	5	Break pixel into this many sub-pixels.
Clean	Yes, 1	Star detection without nearby stars. Strength of cleaning.
Deblending Params		
Deblend Thresh	32	Threshold detection multiplier for background.
Min. Cont.	0.005 – 0.02	Min contrast to identify overlapping stars.
Conv Filter	Gaussian	The convolution filter to apply. This works better on donuts than Default.
Conv FWHM	3	The FWHM (in pixels). Set to seeing.
Photometry Params		
Kron Factor	N/A	Used for ellipse detection – not used for focus.
Shape	Circle	Circle bypasses kron usage and uses r_min
r_min	5	Aperture size (in pixels) for photometry.
Filtering Params		
InitialKeep	1000	Stop looking for stars after this many. Reduce if process is cpu bound.
Max Size	0	0 to ignore this parameter.
Min Size	0	0 to ignore this parameter.
Max Ellipse	1.5	Stop ellipses being counted as stars, e.g. galaxies
stars keep #	0	0 to ignore this parameter.
Cut Brightest	0	0 to ignore this parameter.
Cut Dimmest	10	Filter out dimmest stars which could be just noise.
Sat. Limit	90	Exclude saturated stars.



Adaptive Focus

Adaptive focus will adapt the focuser position between Autofocus runs, aiming to keep optimum focus for each sub-frame. Adaptations for Temperature and Altitude are supported. Further dimensions could be added in future. As confidence in the utility grows less Autofocusing should be required allowing less time for focusing and more time imaging.

The relationship between temperature and focus is likely to be complex. There is some information on this on the internet. Broadly there is a physical effect of temperature on the telescope that expands and contracts as temperature changes as well as a change in refractive index of materials used in lenses as well as air with temperature. These and more effects combine to change the point of best focus with temperature.

The altitude effect is likely to be less than for temperature and may only be noticeable on equipment that is very sensitive to focus.

This is a “pipes and plumbing” release to get things working and builds on extensive work already completed in Ekos. It is experimental in nature and is probably best used in addition to existing re-focus techniques.

To setup, set the Ticks / °C and/or Ticks / °Alt fields in the Filter Settings popup for the filters you want adaptive focusing to work for. The temperature and Altitude of the Last AF Run is shown for reference. In this example, we are going to adapt for temperature only.

Filter Settings

Filter Wheel **Filter Simulator**

	Filter	Exposure	Offset	Auto Focus	Lock Filter	Last AF Solution	Last AF Temp	Last AF Alt	Ticks / °C	Ticks / °Alt	Wavelength
1	Lum	1	300	<input checked="" type="checkbox"/>	--	36580	10	60.3744	9	0	350
2	Red	2	13	<input checked="" type="checkbox"/>	Lum	36759	0	51.5778	0	0	500
3	Green	3	-130	<input checked="" type="checkbox"/>	Lum	36752	0	51.5778	0	0	500
4	Blue	1	-168	<input checked="" type="checkbox"/>	Lum	36591	0	51.5778	0	0	500
5	Sii	5	-86	<input checked="" type="checkbox"/>	Lum	36773	0	51.5778	0	0	500
6	Ha	5	-51	<input checked="" type="checkbox"/>	Lum	36692	0	51.5778	0	0	500
7	Oiii	5	2	<input checked="" type="checkbox"/>	Lum	36652	0	51.5778	0	0	500
8		0.001	0	<input type="checkbox"/>	--	36651			0	0	500

Set *relative* filter offsets in steps either directly in the table or use the Build Offsets tool. Build Offsets

Capture flats at the same focus as lights Close

You will need to work out Ticks / °C and/or Ticks / °Alt for your equipment. To do this there is an existing utility in Ekos whereby when Focus logging is enabled, in addition to adding focus messages to the debug log, every time an Autofocus run completes information is written to a text file in a directory called focuslogs located in the same place as the debug logs directory. The files are called "autofocus-<datetime>.txt". The data written are: date, time, position, temperature, filter, HFR, altitude. This data will need to be analysed outside of Ekos to determine the Ticks / °C and if required the Ticks / °Alt. This release will support a simple linear relationship between temperature, or altitude, and ticks. In the future, if there is demand, more sophisticated relationships could be supported.

A way to get a value for Ticks / °C would be to take the data from the autofocus-<datetime>.txt files from a few nights of observing into a spreadsheet and graph focus position against temperature for each filter. Review the data and remove any outliers and plot a line of best fit. Use the line to get Ticks / °C. If you intend to adapt for altitude as well as temperature, then it would be better to use a set of data at similar altitudes when calibrating temperature. You will need to ensure that your focus position is repeatable at the same temperature and altitude and that there is no slipping of the focuser or uncompensated backlash.

If you believe that your equipment would benefit from adaptive focusing on altitude, then you could perform autofocus at a range of altitudes at the same temperature and repeat the graphing exercise outlined above.

You will need a temperature source in order to adapt on temperature.

On the Focus screen, there is a new checkbox in Settings called "Adaptive Focus". Check to activate the feature.

I ran Autofocus and it solved at 36580 and 10°C (as per the Filter Settings popup above). Then I ran a simple sequence of 5 subframes. I adjusted the temperature firstly to 9°C then to 8°C. After each subframe completes, Ekos performs an adaptive focus run and where there is a temperature change it calculates the number of ticks to move the focuser. In this example, the focuser was moved inward by 9 ticks on 2 separate occasions, starting at 36580, before moving to 36571 and then to 36562 as shown on the Focus Tab in the Current Position widget and in the message box.

The screenshot displays the Ekos - Simulators Profile interface. Key elements include:

- Focuser Section:** Shows 'Focuser: Focuser Simulator' and 'Steps: 36562' and '36580'.
- Camera & Filter Wheel Section:** Shows 'TS: Focuser Simulator' and 'Filter: Lum'. Below this, it indicates 'Temp. = 8.00 °C' and 'ΔT = -2.00 °C'.
- Settings Section:** Under the 'Adaptive Focus' tab, 'Adaptive Focus' is checked, and 'Min Move: 1 ticks' is set.
- Star Field Image:** A dark image showing several stars with numerical values (e.g., 1.02, 1.01, 0.98) overlaid in red.
- V-Curve Graph:** A plot of HFR (pix) vs. position (ticks). The curve is labeled 'LTP: Hyperbola (W) Solution: 36580, R²=0.99'. The x-axis ranges from 25000 to 45000, and the y-axis ranges from 0.75 to 2.0. Points 1 through 7 are marked on the curve.
- Message Log:** A log at the bottom shows focus adjustments:
 - 2023-02-09T18:28:35 Focusing inward by 9 steps...
 - 2023-02-09T18:28:35 Adaptive Focus moving from 36,571 to 36,562
 - 2023-02-09T18:28:35 Adaptive Focus: Temp delta -9 ticks; Alt delta 0 ticks
 - 2023-02-09T18:28:14 Adaptive Focus: Temp delta 0 ticks; Alt delta 0 ticks
 - 2023-02-09T18:27:53 Adaptive Focus: Temp delta 0 ticks; Alt delta 0 ticks
 - 2023-02-09T18:27:32 Focusing inward by 9 steps...

There are 2 controls on Adaptive Focus:

1. Min Move. Only move the focuser if the movement is equal to or above this number of ticks. Designed to stop very small moves of no benefit.
2. Max Total Move. Deadman's handle on Adaptive Focus. If Adaptive Focus attempts to move the focuser by more than this amount Adaptive Focus is deactivated (until reset by the user) but any unattended schedule will continue but without Adaptive Focus.

AF Start

The AF Start feature will adapt the starting position of a Linear 1 Pass Autofocus Run appropriately for the filter selected and current temperature and altitude. It relies on Adaptive Focus data being setup as detailed in the Fitting Measure section.

The purpose of this feature is to start the AF run as close as possible to the actual focus point which will allow fixed step walks as detailed in the Walk section to work correctly. By starting an AF Run as close as possible to the point of focus, the solution found will be as accurate as possible.

To activate the feature, check the “AF Start” checkbox.

Here is an example. An AF Run on the Red filter has been done at 36711 and 5°C. Note that the last successful Lum filter AF Run was at 36498 and 5°C and that the Lum filter Ticks / °C is set to 9.

The screenshot shows the Ekos - Simulators Profile window. The Focuser section is set to 'Focuser Simulator' with 'Steps' at 36711. The Camera & Filter Wheel section shows 'Temp. = 5.00 °C' and 'ΔT = 0 °C'. The Filter Wheel section is set to 'Red'. A 'Filter Settings' dialog box is open, showing a table of filter settings.

Filter	Exposure	Offset	Auto Focus	Lock Filter	Last AF Solution	Last AF Temp	Last AF Alt	Ticks / °C	Ticks / °A
1 Lum	1	0	<input checked="" type="checkbox"/>	--	36498	5	51.5778	9	0
2 Red	2	0	<input checked="" type="checkbox"/>	--	36711	5	51.5778	0	0

Below the table, there is a message: "Set relative filter offsets in steps either directly in the table or use the Build Offsets tool." and a checkbox for "Capture flats at the same focus as lights".

So now we do an AF Run on Lum with Adapt Start Pos checked. Notice that the temperature is now 0°C. So, Ekos will take the last AF Run on Lum = 36498 and adapt the temperature by $-5^{\circ}\text{C} * 9 \text{ ticks} = 36498 - 45 = 36453$. This is shown in the message window.

The screenshot displays the Ekos - Simulators Profile software interface. The main window is titled "Ekos - Simulators Profile" and features a toolbar with various icons. The interface is organized into several sections:

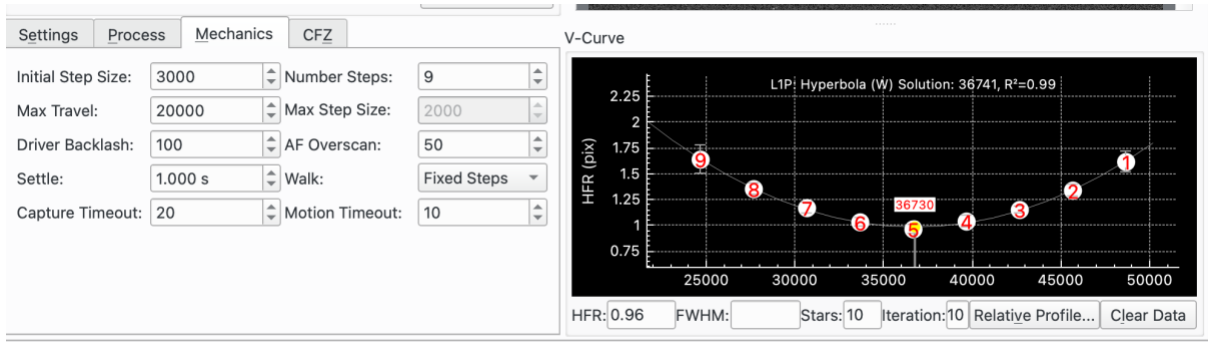
- Focuser:** Shows "Focuser Simulator" with "Steps" set to 36572. It includes buttons for "Auto Focus" and "Stop".
- Camera & Filter Wheel:** Displays "Exp: 1.000", "Bin: 1x1", "Gain: 100.00", and "Filter: Lum". A red box highlights "Temp. = 0.00 °C" and "ΔT = 0 °C".
- Settings:** Includes tabs for "Process", "Mechanics", and "CFZ". Under "Mechanics", "Adapt Start Pos" is checked and highlighted with a red box.
- V-Curve:** A plot showing "HFR (pix)" vs. "Steps" with a hyperbola fit. The plot title is "L1P: Hyperbola (W) Solution: 36572, R²=0.99". Data points are labeled 1 through 6.
- Status Bar:** Shows a log of focus adjustments, with "2023-02-11T10:59:04 Adapting start point from 36,711 to 36,453" highlighted in red.

Walk

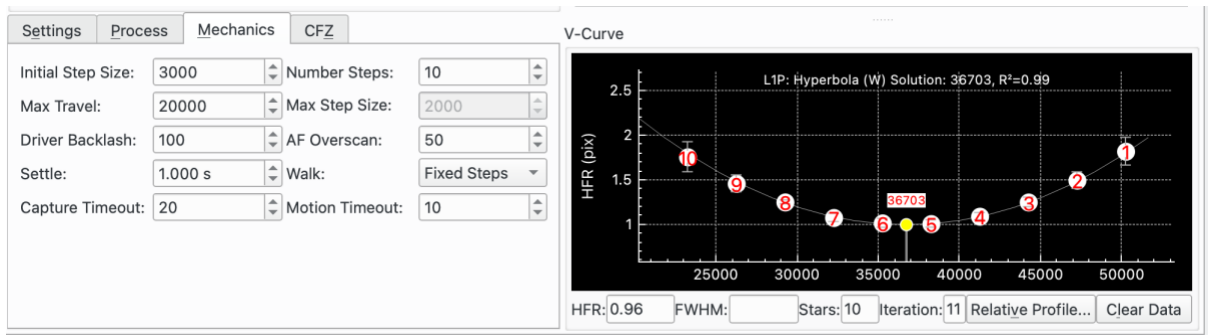
In addition to the current "Classic" walk, 2 new walks have been added. These are available as a dropdown box in the Mechanics tab. Both of these are intended to be used when starting an Autofocus run very close to focus, for example during a session where focus has already been found. They allow for a symmetric curve to be drawn with the same number of points on either side of the minimum (or maximum) and give the user complete control over the number of points.

Fixed Steps

Allows a fixed number of steps to be specified. The "Out Step Multiple" field is replaced by "Number Steps" to allow a value to be entered. Specifying an "odd" number of points allows the centre point to be placed very close to the minimum.

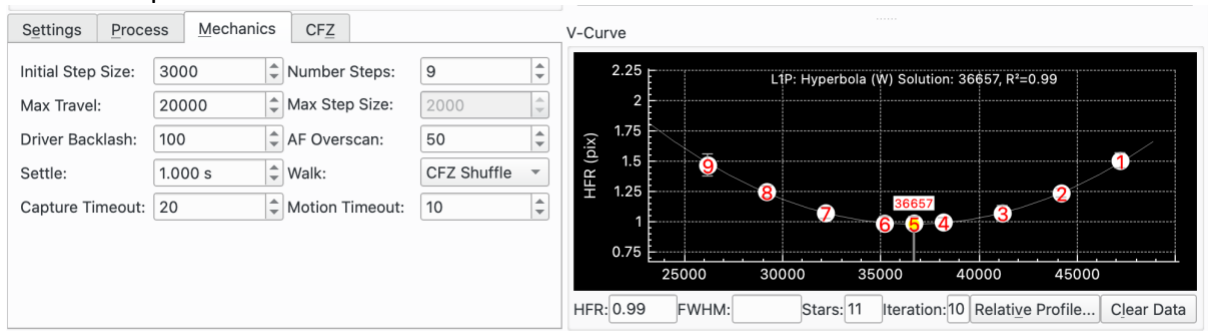


Specifying an “even” number of points allows the 2 centre points to be placed either side of the minimum.

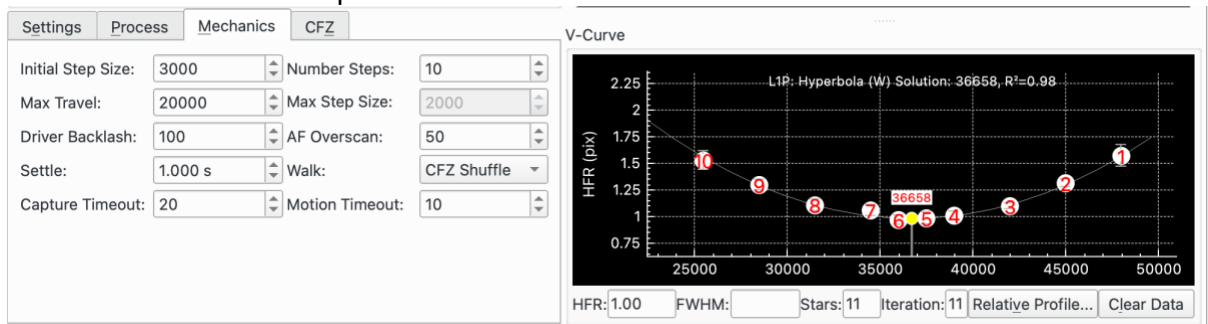


CFZ Shuffle

This is similar to “Fixed Steps” but takes half-steps around the minimum. For an “odd” number of points:



For an “even” number of points:

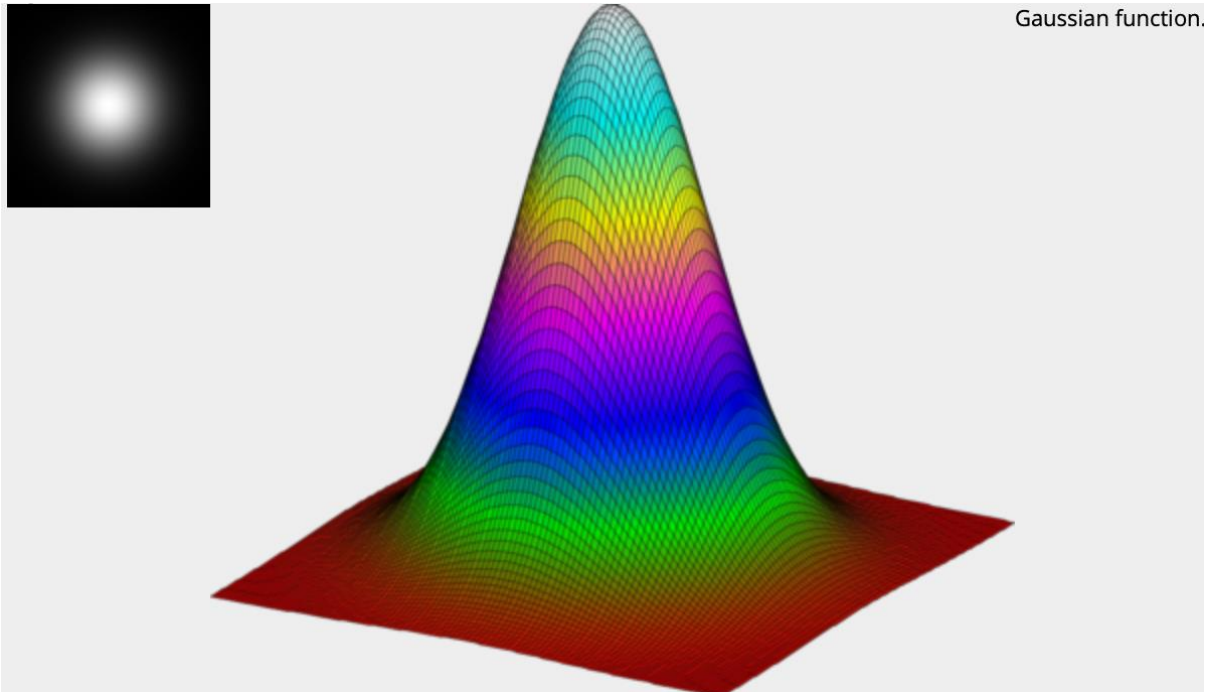


Overscan

Overscan was originally built for the Linear focus algorithm, and then incorporated into Linear 1 Pass. It is an alternative to Driver Backlash and can be used with focusers whose drivers do not support backlash. If backlash is, for example, between 90 and 100 ticks then an AF Overscan value > 100 ticks (e.g. 150 ticks) can be set that will neutralise backlash.

Overscan has been extended to all outward focus movements: all focus algorithms, all manual focus movements, Adaptive Focus, AF Start, and "Take Flats at Same Position as Lights".

Appendix 1 – FWHM



The general equation of a two dimensional Gaussian takes the form:

$$f(x, y) = b + a \cdot e^{-\{A \cdot (x-x_0)^2 + 2B \cdot (x-x_0) \cdot (y-y_0) + C \cdot (y-y_0)^2\}} \quad (1)$$

Where:

b = background

a = amplitude

x_0 = x position of the maximum

y_0 = y position of the maximum

A, B and C are parameters forming the quadratic exponent.

There are some optimisations that could be made here such as:

- Background b, is calculated by SEP so could use that.
- Amplitude a, is calculated by SEP so could use that.
- Centroid x_0 and y_0 are calculated by SEP so could use these.
- Could assume spherical symmetry which would effectively remove B

In this release none of these optimisations have been used so that the curve fitting process remains generic.

To make the A, B and C parameters more meaningful lets move from x, y coordinates to polar coordinates r, α where:

$$\begin{aligned} x &= r \cdot \cos \alpha \\ y &= r \cdot \sin \alpha \end{aligned}$$

Now, if the quadratic represents an ellipse rotated at angle θ to the x-axis, then we can rewrite in polar coordinates r, $(\alpha - \theta)$ where:

$$\begin{aligned}x' &= r \cdot \cos(\alpha - \theta) \\y' &= r \cdot \sin(\alpha - \theta)\end{aligned}$$

Expanding these:

$$\begin{aligned}x' &= r \cdot \cos(\alpha - \theta) = r \cdot \cos \alpha \cdot \cos \theta + r \cdot \sin \alpha \cdot \sin \theta \\x' &= x \cdot \cos \theta + y \cdot \sin \theta\end{aligned}$$

$$\begin{aligned}y' &= r \cdot \sin(\alpha - \theta) = r \cdot \sin \alpha \cdot \sin \theta - r \cdot \cos \alpha \cdot \sin \theta \\y' &= y \cdot \cos \theta - x \cdot \sin \theta\end{aligned}$$

Returning to equation (1). We have chosen coordinates (x', y') aligned with the principal axes of the ellipse. In addition, if we shift the origin to match the centroid, then

$$f(x', y') = b + a \cdot e^{-\left\{\frac{x'^2}{2\sigma_x^2} + \frac{y'^2}{2\sigma_y^2}\right\}} \quad (2)$$

So, expanding (2):

$$\begin{aligned}\frac{x'^2}{2\sigma_x^2} + \frac{y'^2}{2\sigma_y^2} &= \frac{x^2 \cos^2 \theta + 2xy \cos \theta \sin \theta + y^2 \sin^2 \theta}{2\sigma_x^2} \\&+ \frac{x^2 \sin^2 \theta - 2xy \cos \theta \sin \theta + y^2 \cos^2 \theta}{2\sigma_y^2}\end{aligned}$$

Equating (1) and (2) after translating the coordinate origin in (1), we get:

$$\begin{aligned}A &= \frac{\cos^2 \theta}{2\sigma_x^2} + \frac{\sin^2 \theta}{2\sigma_y^2} \\B &= \frac{\cos \theta \sin \theta}{2\sigma_x^2} - \frac{\cos \theta \sin \theta}{2\sigma_y^2} \\C &= \frac{\sin^2 \theta}{2\sigma_x^2} + \frac{\cos^2 \theta}{2\sigma_y^2}\end{aligned}$$

Now to get expressions for θ , σ_x and σ_y .

Notice:

$$A - C = \frac{\cos^2 \theta - \sin^2 \theta}{2\sigma_x^2} - \frac{\cos^2 \theta - \sin^2 \theta}{2\sigma_y^2} = \frac{\cos 2\theta}{2\sigma_x^2} - \frac{\cos 2\theta}{2\sigma_y^2}$$

And:

$$2B = \frac{2\cos \theta \sin \theta}{2\sigma_x^2} - \frac{2\cos \theta \sin \theta}{2\sigma_y^2} = \frac{\sin 2\theta}{2\sigma_x^2} - \frac{\sin 2\theta}{2\sigma_y^2}$$

So:

$$\frac{2B}{A - C} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

$$\theta = \frac{1}{2} \tan^{-1} \left\{ \frac{2B}{A-C} \right\} \quad (4)$$

Now, going back to the equations A, B and C above. If we take $A \cdot \cos^2 \theta$, $2B \cdot \cos \theta \sin \theta$, and $C \cdot \sin^2 \theta$ then the $2\sigma_y^2$ terms cancel out leaving:

$$A \cdot \cos^2 \theta + 2B \cdot \cos \theta \sin \theta + C \cdot \sin^2 \theta = \frac{1}{2\sigma_x^2} \{ \cos^4 \theta + \cos^2 \theta \sin^2 \theta + \sin^4 \theta \}$$

$$A \cdot \cos^2 \theta + 2B \cdot \cos \theta \sin \theta + C \cdot \sin^2 \theta = \frac{1}{2\sigma_x^2} \{ \cos^2 \theta + \sin^2 \theta \}^2 = \frac{1}{2\sigma_x^2}$$

So:

$$\sigma_x^2 = \frac{1}{2 \cdot \{ A \cdot \cos^2 \theta + 2B \cdot \cos \theta \sin \theta + C \cdot \sin^2 \theta \}} \quad (5)$$

Similarly, notice that if we take $A \cdot \sin^2 \theta$, $-2B \cdot \cos \theta \sin \theta$, and $C \cdot \cos^2 \theta$ then the $2\sigma_x^2$ terms cancel out leaving:

$$A \cdot \sin^2 \theta - 2B \cdot \cos \theta \sin \theta + C \cdot \cos^2 \theta = \frac{1}{2\sigma_y^2} \{ \sin^4 \theta + \cos^2 \theta \sin^2 \theta + \cos^4 \theta \}$$

$$A \cdot \sin^2 \theta - 2B \cdot \cos \theta \sin \theta + C \cdot \cos^2 \theta = \frac{1}{2\sigma_y^2} \{ \sin^2 \theta + \cos^2 \theta \}^2 = \frac{1}{2\sigma_y^2}$$

$$\sigma_y^2 = \frac{1}{2 \cdot \{ A \cdot \sin^2 \theta - 2B \cdot \cos \theta \sin \theta + C \cdot \cos^2 \theta \}} \quad (6)$$

So, equations (4), (5) and (6) allow us to convert from A, B and C to θ , σ_x and σ_y which have more physical meaning.

Returning to Levensburg-Marquardt we need to calculate the Jacobian J, by differentiating (1) with respect to the parameters (b, a, x_0 , y_0 , A, B, C):

$$Jacobian J = \left\{ \frac{\partial f}{\partial b}, \frac{\partial f}{\partial a}, \frac{\partial f}{\partial x_0}, \frac{\partial f}{\partial y_0}, \frac{\partial f}{\partial A}, \frac{\partial f}{\partial B}, \frac{\partial f}{\partial C} \right\}$$

$$\frac{\partial f}{\partial b} = 1$$

$$\frac{\partial f}{\partial a} = \varphi$$

Where:

$$\varphi = e^{-A \cdot (x-x_0)^2 + 2B \cdot (x-x_0) \cdot (y-y_0) + C \cdot (y-y_0)^2}$$

$$\frac{\partial f}{\partial x_0} = [2A(x - x_0) + 2B(y - y_0)] \cdot a \cdot \varphi$$

$$\frac{\partial f}{\partial y_0} = [2C(y - y_0) + 2B(x - x_0)] \cdot a \cdot \varphi$$

$$\frac{\partial f}{\partial A} = -(x - x_0)^2 \cdot a \cdot \varphi$$

$$\frac{\partial f}{\partial B} = -2 \cdot (x - x_0) \cdot (y - y_0) \cdot a \cdot \varphi$$

$$\frac{\partial f}{\partial C} = -(y - y_0)^2 \cdot a \cdot \varphi$$

In order to use the geodesic acceleration optimisation it is necessary to calculate the second partial differentials:

$$\begin{array}{ccccccc}
 \frac{\partial^2 f}{\partial b^2} & \frac{\partial^2 f}{\partial b \partial a} & \frac{\partial^2 f}{\partial b \partial x_0} & \frac{\partial^2 f}{\partial b \partial y_0} & \frac{\partial^2 f}{\partial b \partial A} & \frac{\partial^2 f}{\partial b \partial B} & \frac{\partial^2 f}{\partial b \partial C} \\
 \frac{\partial^2 f}{\partial a \partial b} & \frac{\partial^2 f}{\partial a^2} & \frac{\partial^2 f}{\partial a \partial x_0} & \frac{\partial^2 f}{\partial a \partial y_0} & \frac{\partial^2 f}{\partial a \partial A} & \frac{\partial^2 f}{\partial a \partial B} & \frac{\partial^2 f}{\partial a \partial C} \\
 \frac{\partial^2 f}{\partial x_0 \partial b} & \frac{\partial^2 f}{\partial x_0 \partial a} & \frac{\partial^2 f}{\partial x_0^2} & \frac{\partial^2 f}{\partial x_0 \partial y_0} & \frac{\partial^2 f}{\partial x_0 \partial A} & \frac{\partial^2 f}{\partial x_0 \partial B} & \frac{\partial^2 f}{\partial x_0 \partial C} \\
 \frac{\partial^2 f}{\partial y_0 \partial b} & \frac{\partial^2 f}{\partial y_0 \partial a} & \frac{\partial^2 f}{\partial y_0 \partial x_0} & \frac{\partial^2 f}{\partial y_0^2} & \frac{\partial^2 f}{\partial y_0 \partial A} & \frac{\partial^2 f}{\partial y_0 \partial B} & \frac{\partial^2 f}{\partial y_0 \partial C} \\
 \frac{\partial^2 f}{\partial A \partial b} & \frac{\partial^2 f}{\partial A \partial a} & \frac{\partial^2 f}{\partial A \partial x_0} & \frac{\partial^2 f}{\partial A \partial y_0} & \frac{\partial^2 f}{\partial A^2} & \frac{\partial^2 f}{\partial A \partial B} & \frac{\partial^2 f}{\partial A \partial C} \\
 \frac{\partial^2 f}{\partial B \partial b} & \frac{\partial^2 f}{\partial B \partial a} & \frac{\partial^2 f}{\partial B \partial x_0} & \frac{\partial^2 f}{\partial B \partial y_0} & \frac{\partial^2 f}{\partial B \partial A} & \frac{\partial^2 f}{\partial B^2} & \frac{\partial^2 f}{\partial B \partial C} \\
 \frac{\partial^2 f}{\partial C \partial b} & \frac{\partial^2 f}{\partial C \partial a} & \frac{\partial^2 f}{\partial C \partial x_0} & \frac{\partial^2 f}{\partial C \partial y_0} & \frac{\partial^2 f}{\partial C \partial A} & \frac{\partial^2 f}{\partial C \partial B} & \frac{\partial^2 f}{\partial C^2}
 \end{array}$$

The first thing to notice is that the matrix is symmetric about the diagonal, for example:

$$\frac{\partial^2 f}{\partial a \partial b} = \frac{\partial^2 f}{\partial b \partial a}$$

So, there are 7 diagonal elements. The 42 off diagonal elements simplify to 21 pairs.

Row 1:

$$\frac{\partial^2 f}{\partial b^2} = \frac{\partial^2 f}{\partial b \partial a} = \frac{\partial^2 f}{\partial b \partial x_0} = \frac{\partial^2 f}{\partial b \partial y_0} = \frac{\partial^2 f}{\partial b \partial A} = \frac{\partial^2 f}{\partial b \partial B} = \frac{\partial^2 f}{\partial b \partial C} = 0$$

Row 2:

$$\frac{\partial^2 f}{\partial a^2} = 0$$

$$\frac{\partial^2 f}{\partial a \partial x_0} = [2A(x - x_0) + 2B(y - y_0)] \cdot \varphi$$

$$\frac{\partial^2 f}{\partial a \partial y_0} = [2C(y - y_0) + 2B(x - x_0)]. \varphi$$

$$\frac{\partial^2 f}{\partial a \partial A} = -(x - x_0)^2. \varphi$$

$$\frac{\partial^2 f}{\partial a \partial B} = -2(x - x_0). (y - y_0). \varphi$$

$$\frac{\partial^2 f}{\partial a \partial C} = -(y - y_0)^2. \varphi$$

Row 3:

$$\frac{\partial^2 f}{\partial x_0^2} = -2A. a. \varphi + [2A(x - x_0) + 2B(y - y_0)]^2. a. \varphi$$

$$\frac{\partial^2 f}{\partial x_0 \partial y_0} = -2B. a. \varphi + [2A(x - x_0) + 2B(y - y_0)]. [-2B(x - x_0) - 2C(y - y_0)]. a. \varphi$$

$$\frac{\partial^2 f}{\partial x_0 \partial A} = 2(x - x_0). a. \varphi - [2A(x - x_0) + 2B(y - y_0)]. (x - x_0)^2. a. \varphi$$

$$\frac{\partial^2 f}{\partial x_0 \partial B} = 2(y - y_0). a. \varphi - [2A(x - x_0) + 2B(y - y_0)]. 2(x - x_0). (y - y_0). a. \varphi$$

$$\frac{\partial^2 f}{\partial x_0 \partial C} = [2A(x - x_0) + 2B(y - y_0)]. (y - y_0)^2. a. \varphi$$

Row 4:

$$\frac{\partial^2 f}{\partial y_0^2} = -2C. a. \varphi + [2C(y - y_0) + 2B(x - x_0)]^2. a. \varphi$$

$$\frac{\partial^2 f}{\partial y_0 \partial A} = -[2C(y - y_0) + 2B(x - x_0)]. (x - x_0)^2. a. \varphi$$

$$\frac{\partial^2 f}{\partial y_0 \partial B} = 2(x - x_0). a. \varphi - [2C(y - y_0) + 2B(x - x_0)]. 2(x - x_0). (y - y_0). a. \varphi$$

$$\frac{\partial^2 f}{\partial y_0 \partial C} = 2(y - y_0). a. \varphi - [2C(y - y_0) + 2B(x - x_0)]. (y - y_0)^2. a. \varphi$$

Row 5:

$$\frac{\partial^2 f}{\partial A \partial B} = 2. (x - x_0)^3. (y - y_0). a. \varphi$$

$$\frac{\partial^2 f}{\partial A \partial C} = (x - x_0)^2 \cdot (y - y_0)^2 \cdot a \cdot \varphi$$

Row 6:

$$\frac{\partial^2 f}{\partial B^2} = 4 \cdot (x - x_0)^2 \cdot (y - y_0)^2 \cdot a \cdot \varphi$$

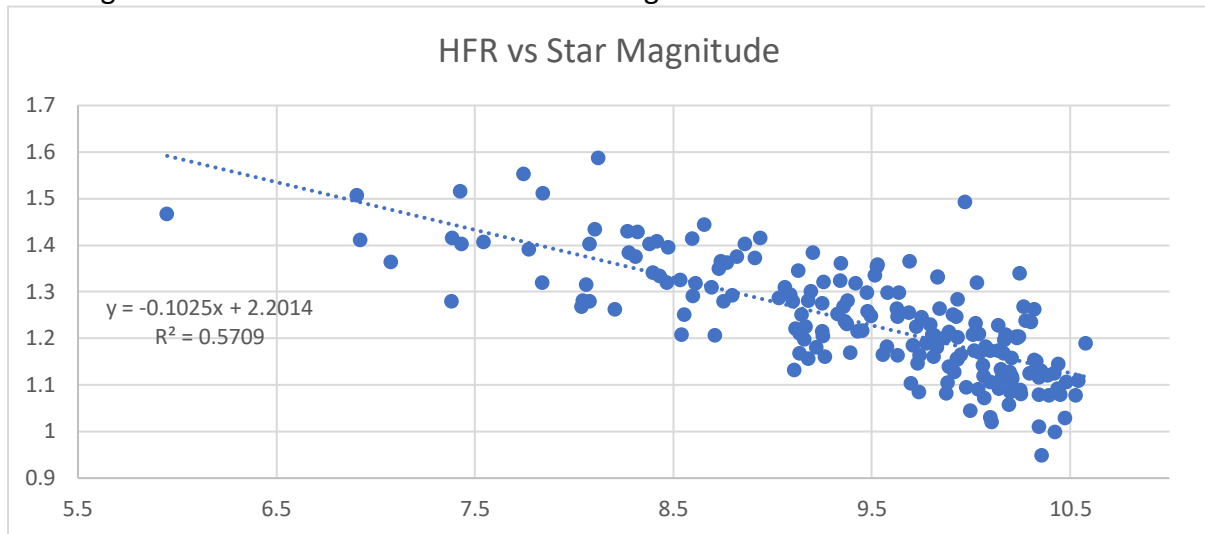
$$\frac{\partial^2 f}{\partial B \partial C} = 2 \cdot (x - x_0) \cdot (y - y_0)^3 \cdot a \cdot \varphi$$

Row 7:

$$\frac{\partial^2 f}{\partial C^2} = (y - y_0)^4 \cdot a \cdot \varphi$$

Appendix 2 – Star Magnitude Compensation

There is a correlation between HFR and star magnitude. This is a plot of an in-focus frame showing a definite trend to lower HFR for less bright stars.

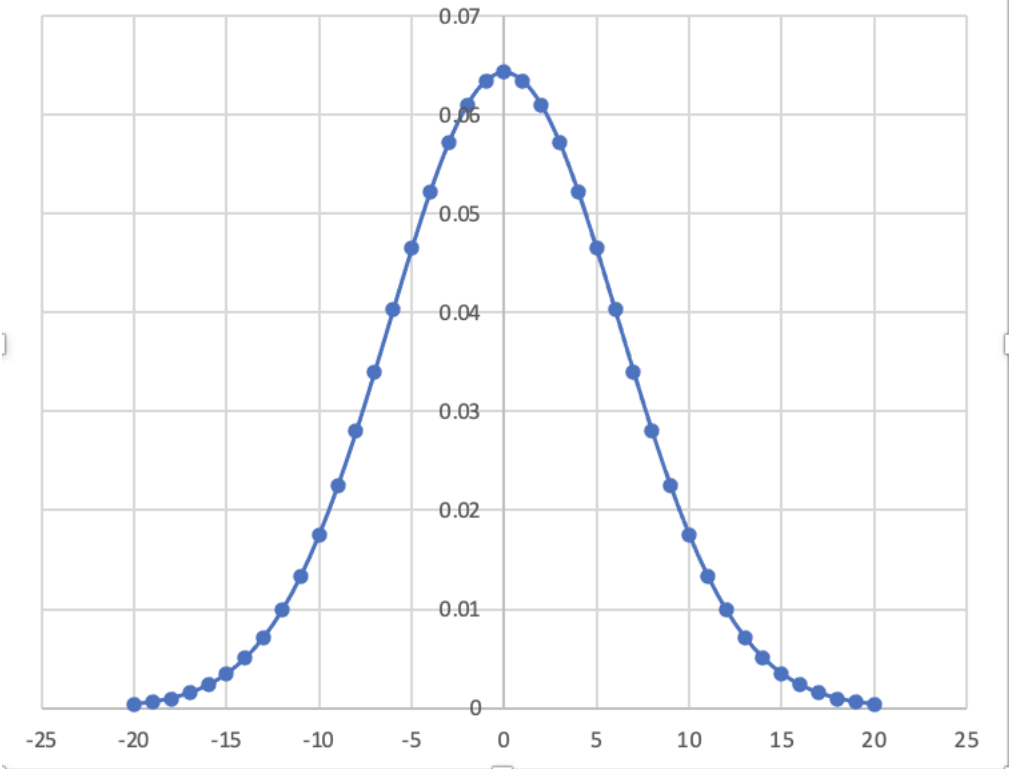


This is probably because of background noise distorting the HFR calculation more for fainter stars than for brighter ones, where only the tip of Airy Disc projects above the background. Each star gets fainter the more out of focus it is, until it drops into the background and disappears (HFR=0).

The same effect occurs for out of focus frames with a similar gradient. SEP background subtracts all data in the focus image as a key step. This distorts the HFR measurements.

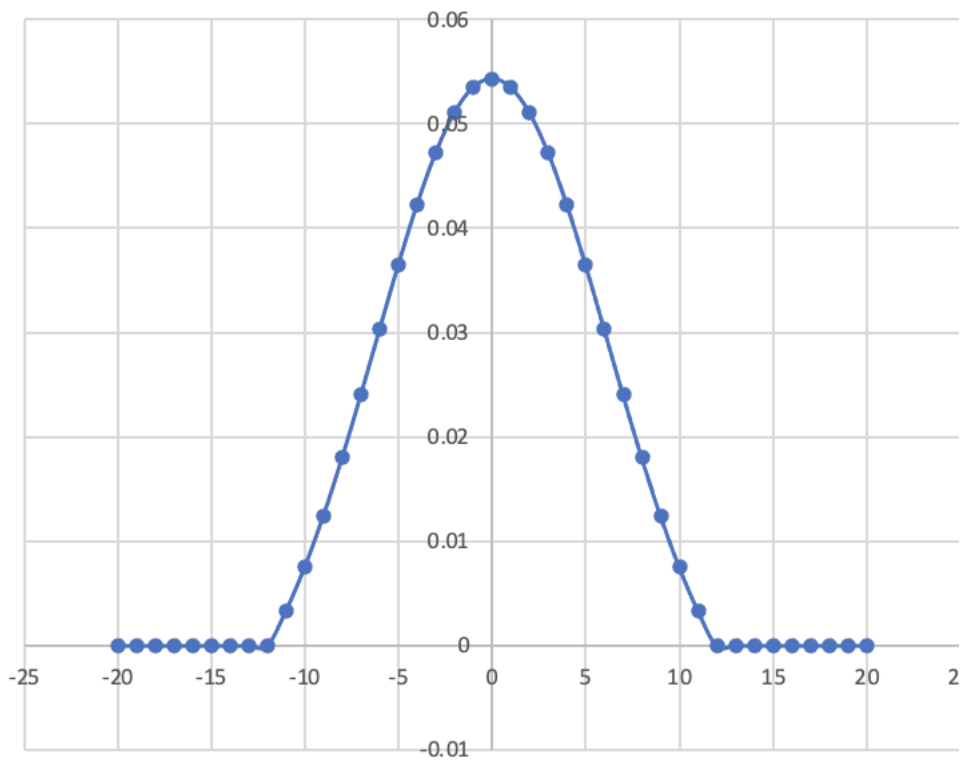
Lets start by assuming that imaging is seeing limited so that we can assume that the star intensity follows a normal (Gaussian) distribution. Don't worry about the units, they aren't important.

Normal Distribution



Now, let's say the background level is 0.01. So after background subtraction we get:

Normal Distribution after Background Subtraction



We can simplify the maths assuming symmetry in x,y and work in 1 dimension. The equation for a normal distribution centred on the origin is:

$$f(x) = a \cdot e^{\frac{-x^2}{2\sigma^2}}$$

Where a is the peak and σ is the standard deviation.

The half flux radius (HFR) is the radius of a circle around the star centre which contains half of the star flux. The other half of the flux is outside this circle. So, using this definition:

$$HFR = \frac{(1)}{(2)} = \frac{\int_0^\infty a \cdot x \cdot e^{\frac{-x^2}{2\sigma^2}} dx}{\int_0^\infty a \cdot e^{\frac{-x^2}{2\sigma^2}} dx} = \frac{\int_0^\infty x \cdot e^{\frac{-x^2}{2\sigma^2}} dx}{\int_0^\infty e^{\frac{-x^2}{2\sigma^2}} dx}$$

The numerator (1) becomes:

$$\left[\frac{-2\sigma^2}{2} \cdot e^{\frac{-x^2}{2\sigma^2}} + C \right]_0^\infty = \sigma^2$$

The denominator (2):

$$\int_0^\infty e^{\frac{-x^2}{2\sigma^2}} dx$$

This is comparable to the standard Gaussian integral (https://en.wikipedia.org/wiki/Gaussian_integral):

$$\int_{-\infty}^\infty e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}}$$

So (2) becomes:

$$\int_0^\infty e^{\frac{-x^2}{2\sigma^2}} dx = \frac{1}{2} \cdot \sqrt{\frac{\pi}{1/2\sigma^2}} = \sigma \cdot \frac{\sqrt{2\pi}}{2} = \sigma \cdot \sqrt{\frac{\pi}{2}}$$

So, the HFR becomes:

$$HFR = \frac{(1)}{(2)} = \frac{\sigma^2}{\sigma \cdot \sqrt{\frac{\pi}{2}}} = \sigma \cdot \sqrt{\frac{2}{\pi}} \quad (3)$$

The HFR depends only on σ , not on star brightness. Yet we all know that brighter stars are fatter.

Now, lets look at the background subtracted star. The star peak has been reduced by the background, b but the intensity still follows the normal distribution until $x(b)$ where it reaches zero. So we can write the equation for the measured HFRm, as follows:

$$HFRm = \frac{(4)}{(5)} = \frac{\int_0^{x_b} a \cdot x \cdot e^{\frac{-x^2}{2\sigma^2}} dx}{\int_0^{x_b} a \cdot e^{\frac{-x^2}{2\sigma^2}} dx} = \frac{\int_0^{x_b} x \cdot e^{\frac{-x^2}{2\sigma^2}} dx}{\int_0^{x_b} e^{\frac{-x^2}{2\sigma^2}} dx}$$

At x_b :

$$f(x) = b = a \cdot e^{\frac{-x_b^2}{2\sigma^2}}$$

$$\ln \frac{a}{b} = \frac{x_b^2}{2\sigma^2}$$

$$x_b = \pm \sigma \sqrt{2 \cdot \ln(a/b)} \quad (6)$$

Back to equation (4) the numerator:

$$\left[\frac{-2\sigma^2}{2} \cdot e^{\frac{-x^2}{2\sigma^2}} + C \right]_0^{x_b} = \sigma^2 \cdot \left\{ 1 - e^{\frac{-x_b^2}{2\sigma^2}} \right\}$$

Substituting (6) gives:

$$\sigma^2 \cdot \left\{ 1 - e^{\frac{-x_b^2}{2\sigma^2}} \right\} = \sigma^2 \cdot \left\{ 1 - e^{\frac{-\sigma^2 \cdot 2 \cdot \ln(a/b)}{2\sigma^2}} \right\} = \sigma^2 \cdot \left\{ 1 - e^{\frac{-\sigma^2 \cdot 2 \cdot \ln(a/b)}{2\sigma^2}} \right\} = \sigma^2 \cdot \left\{ 1 - b/a \right\}$$

Returning to the denominator (5):

$$\int_0^{x_b} e^{\frac{-x^2}{2\sigma^2}} dx$$

This is like the Error Function, erf (https://en.wikipedia.org/wiki/Error_function) which can be solved numerically.

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

If we let $t^2 = \frac{-x^2}{2\sigma^2}$, then:

$$x = \pm \sqrt{2} \cdot \sigma \cdot t$$

$$dx = \sqrt{2} \cdot \sigma \cdot dt$$

Returning to equation (5):

$$\int_0^{x_b} e^{\frac{-x^2}{2\sigma^2}} dx = \int_0^{x_b} e^{t^2} \sqrt{2} \cdot \sigma \cdot dt = \sqrt{2} \cdot \sigma \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{2}{\sqrt{\pi}} \int_0^{t_b} e^{-t^2} dt = \sigma \cdot \sqrt{\frac{\pi}{2}} \cdot \text{erf}(t_b)$$

$$x_b = \sqrt{2} \cdot \sigma \cdot t_b$$

Substituting from (6):

$$t_b = \frac{x_b}{\sqrt{2} \cdot \sigma} = \frac{\sigma \sqrt{2 \cdot \ln(a/b)}}{\sqrt{2} \cdot \sigma} = \sqrt{\ln(a/b)}$$

So (5) becomes:

$$\sigma \cdot \sqrt{\frac{\pi}{2}} \cdot \text{erf} \left\{ \sqrt{\ln(a/b)} \right\}$$

Substituting for (4) and (5) in the HFRm equation:

$$\text{HFRm} = \frac{(4)}{(5)} = \frac{\sigma^2 \cdot \{1 - b/a\}}{\sigma \cdot \sqrt{\frac{\pi}{2}} \cdot \text{erf} \left\{ \sqrt{\ln(a/b)} \right\}} = \sigma \cdot \sqrt{\frac{2}{\pi}} \cdot \frac{1 - b/a}{\text{erf} \left\{ \sqrt{\ln(a/b)} \right\}} \quad (7)$$

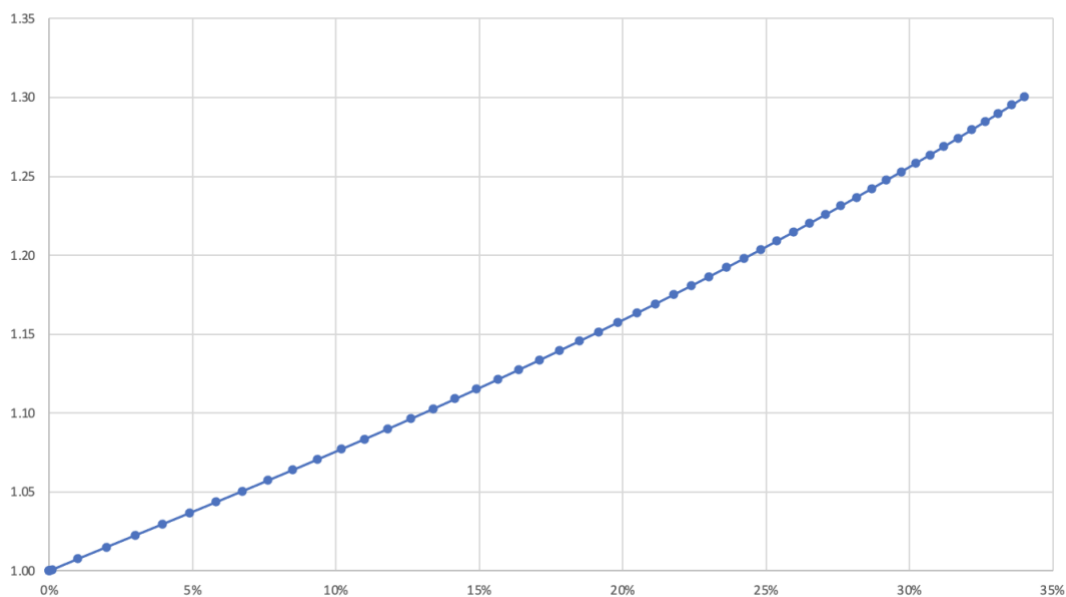
Now we can compare equation 3 (the HFR for the star with no background) with equation (7) (the HFRm for the star with background subtraction):

$$\frac{HFR}{HFRm} = \frac{(3)}{(7)} = \frac{\sigma \cdot \sqrt{\frac{2}{\pi}} \operatorname{erf}\left\{\sqrt{\ln(a/b)}\right\}}{\sigma \cdot \sqrt{\frac{2}{\pi}} \cdot \frac{1 - b/a}{1 - b/a}}$$

$$HFR = HFRm \cdot \frac{\operatorname{erf}\left\{\sqrt{\ln(a/b)}\right\}}{1 - b/a}$$

Looking at how the factor scales the measured HFR against the b/a ratio...

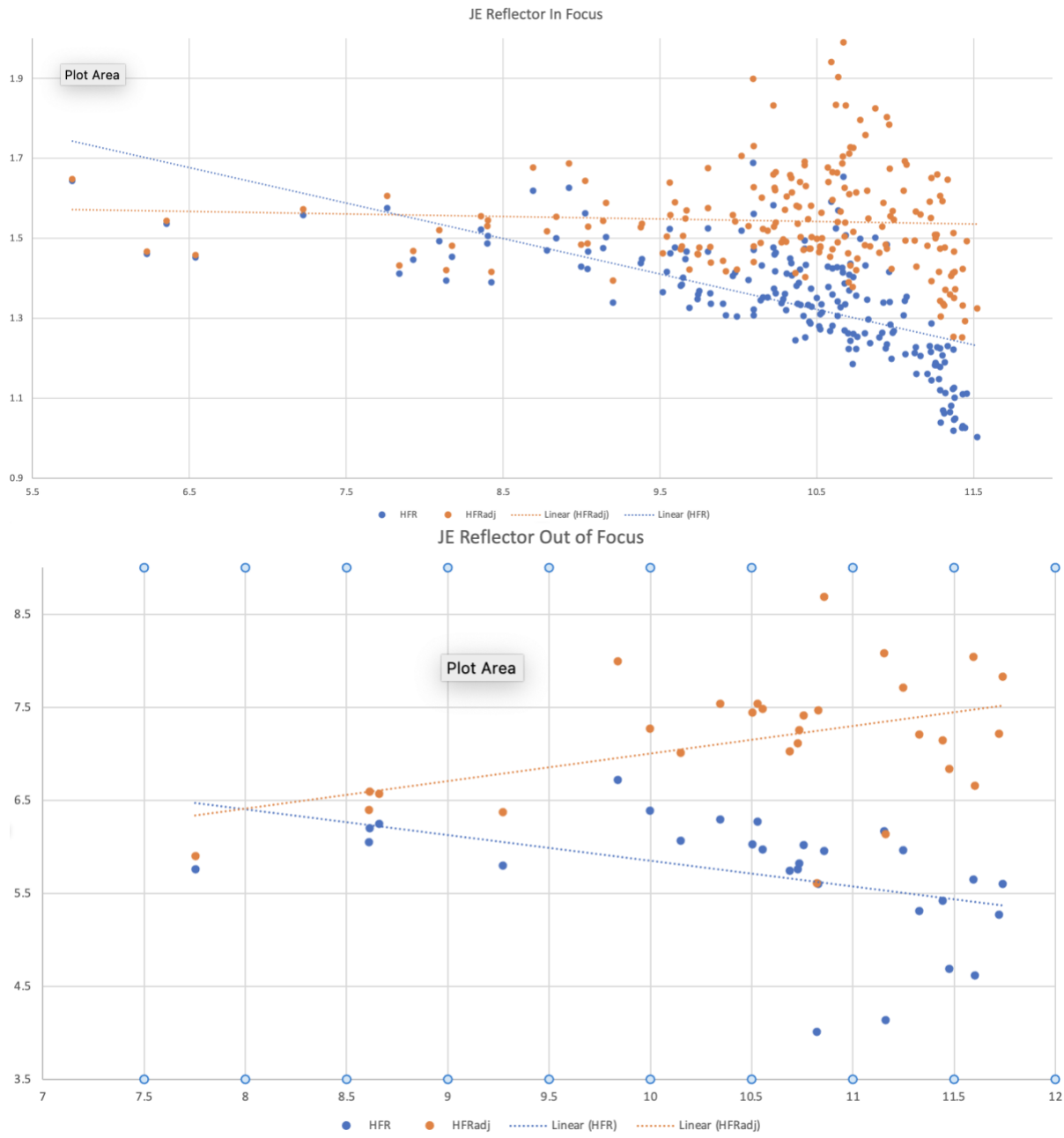
HFR Scaling Factor vs Background Intensity / Peak



As expected, when the background is negligible compared to the star's peak intensity, the factor tends to 1. That is, there is no real adjustment to HFR. As the background becomes larger in relation to a star's peak, so the HFR scaling factor becomes more significant, reducing the observed HFR. This ties in with the observation that fainter stars are thinner and brighter stars are fatter.

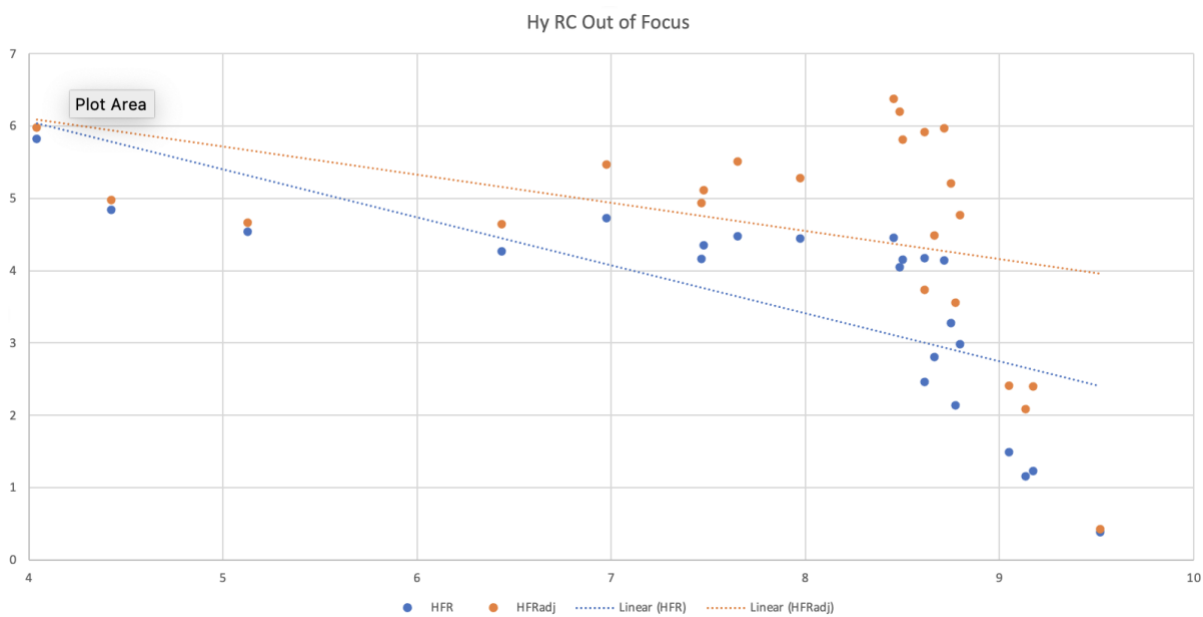
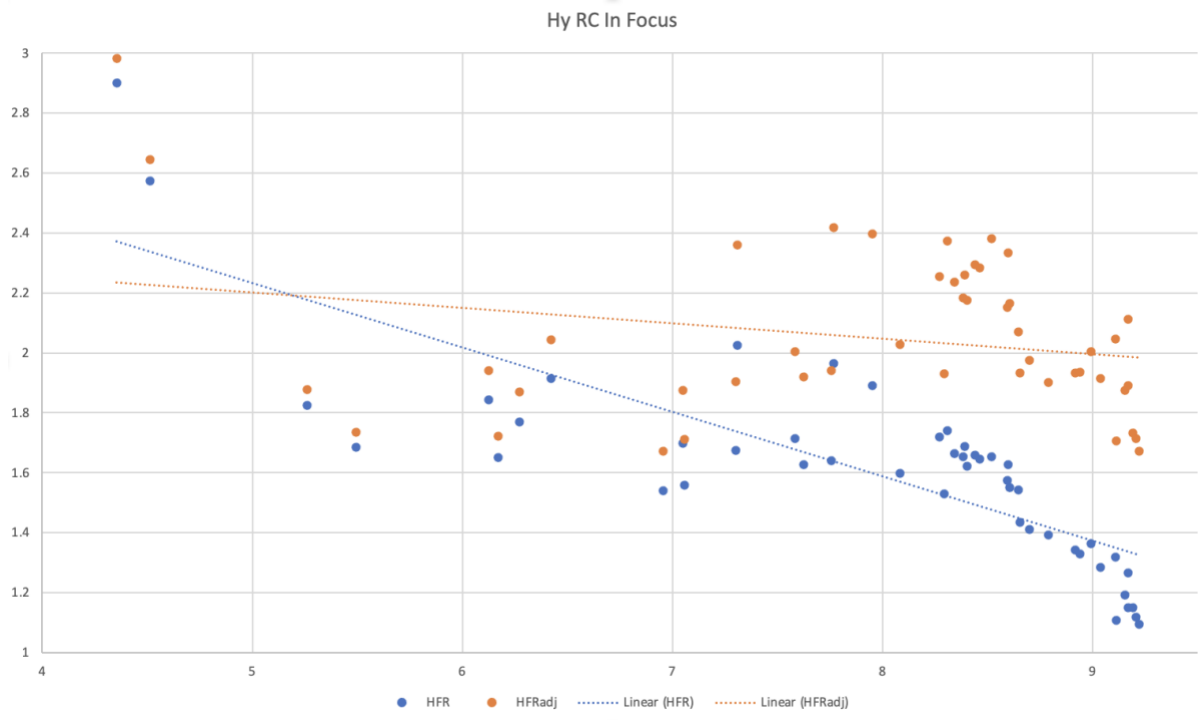
So it is possible to plot the HFR against star magnitude for both the measure HFR and a background subtracted adjusted HFRadj using the formula above.

Case 1 – My Esprit 100mm 550FL Reflector.



For the "In Focus" frame the adjustment has done a good job of flattening the trendline. For the "Out of Focus" frame it appears to have overcompensated and not improved things.

Case 2 – Hy's RC Telescope

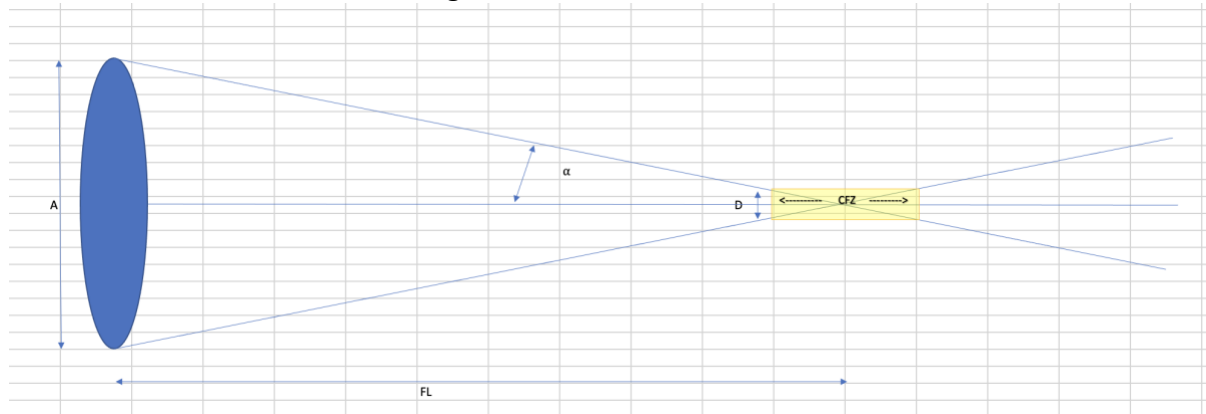


In this case, both the “in focus” and “out of focus” trendlines have been improved by adjusting the HFR.

So it would appear that this approach has some mileage in making stars more comparable in a focus frame. Running stats on stars in a focus frame both bright and faint ought to be more meaningful after the stars’ HFR has been adjusted.

Appendix 3 – Critical Focus Zone (CFZ)

The CFZ is the zone, centred on the point of optimum focus, that focus can be moved within, without a detectable change in focus.



$$\sin \alpha = \frac{A}{2.FL}$$

Where A is the aperture size and FL is the focal length of the telescope.

We know:

$$f = \frac{FL}{A}$$

$$\sin \alpha = \frac{1}{2.f}$$

We know that:

$$\frac{1}{2} CFZ = \frac{D/2}{\tan \alpha}$$

Where D is the size of the disc

$$CFZ = \frac{D}{\tan \alpha} \approx 2.D.f \quad (8)$$

Airy Disc

The Airy disc angular radius is given by:

$$\theta \approx 1.22 \frac{\lambda}{A}$$

Converting to a linear distance at the focal point:

$$\theta.FL \approx 1.22 \frac{\lambda}{A} FL = 1.22\lambda f$$

So

$$D = 2.\theta.FL = 2.44 \lambda f$$

From (1)

$$CFZ = 2 \times 2.44\lambda f.f = 4.88\lambda f^2 \quad (9)$$

Generalising, it is accepted that it is possible to resolve detail finer than the diameter of the Airy Disc. If we replace θ by $t.\theta$ where t is the fraction of the Airy Disc that can be resolved then (2) becomes:

$$CFZ = 4.88t\lambda f^2 \quad (10)$$

Some folks think that $t=1/3$ is a reasonable value. For example, the “In Perfect Focus” article by Don Goldman and Barry Megdal in Sky & Telescope 2010. Obviously, setting t to 1 gives the “standard” equation for CFZ.

Wavefront Aberration

Another way of looking at this is to consider a spherical wave at the aperture centred at the Focal Point, FL and another at the edge of the CFZ, FL'. Consider the sagitta, s

$$s = FL - \sqrt{FL^2 - (A/2)^2}$$

$$s \approx FL - FL \sqrt{1 - \frac{1}{2} \left(\frac{A}{2.FL} \right)^2}$$

Assuming $FL \gg A$, then:

$$s \approx FL - FL \left\{ 1 - \frac{1}{2} \left(\frac{A}{2.FL} \right)^2 \right\} \approx FL \cdot \frac{1}{2} \left(\frac{A}{2.FL} \right)^2$$

$$s \approx \frac{A^2}{8.FL}$$

So, now when we defocus to the edge of the CFZ, s moves to s' and FL moves to FL':

$$s - s' = \frac{A^2}{8.FL} - \frac{A^2}{8.FL'}$$

If we require $s - s'$ to be $n.\lambda$, where n is the number of wavelengths of aberration, then:

$$s - s' = n.\lambda = \frac{A^2}{8} \left\{ \frac{1}{FL} - \frac{1}{FL'} \right\} = \frac{A^2}{8} \left\{ \frac{FL - FL'}{FL.FL'} \right\}$$

Now, we know that $FL' = FL - \frac{1}{2}.CFZ$:

$$n.\lambda \approx \frac{A^2}{8} \left\{ \frac{CFZ}{2.FL^2} \right\}$$

$$\frac{CFZ}{2} \approx \frac{8.n.\lambda.FL^2}{A^2} = 8n\lambda f^2$$

$$CFZ = 4n\lambda f^2 \quad (11)$$

People have suggested using $n=1/3$ and $n=1/10$

Seeing Disc

Another way of looking at this is to consider a seeing disc, θ and to allow a tolerance t , that is unresolvable. Returning to equation (1):

$$CFZ = 2.D.f = 2.t.\theta.f$$

To convert θ to arc – seconds:

$$\frac{360 * 60 * 60}{2\pi} = 206,265$$

Convert to a linear distance at focal point:

$$CFZ = \frac{2}{206,265} . t . \theta . f . FL = \frac{1}{103,133} . t . \theta . f^2 . A$$

If we convert A from m to mm and produce the CFZ result in microns, then:

$$CFZ = \frac{1,000,000}{103,133} . t . \theta . f^2 . \frac{A}{1000} = \frac{1}{103.133} . t . \theta . f^2 . A$$

Let:

$$t = \sqrt{\frac{\tau}{100}} = \frac{\sqrt{\tau}}{10}$$

So that:

$$CFZ = \frac{1}{1031.33} . \sqrt{\tau} . \theta . f^2 . A = 0.0009696 . \sqrt{\tau} . \theta . f^2 . A$$

Gold Astro have performed a more sophisticated analysis of this:

<http://www.goldastro.com/goldfocus/ncfz.php>

Unfortunately, whilst providing some clues by saying they have used Nijboer-Zernike theory, they haven't published how they arrived at their equation.

$$CFZ = 0.00225 . \sqrt{\tau} . \theta . f^2 . A \quad (12)$$

Impact of Pixel Size on Resolution

For astrophotography the theoretical CFZ is one consideration, but another is the resolving capability of the CCD itself. Going back to equation (1):

$$CFZ = 2.D.f$$

In this case, D can be replaced by half the pixel size (so we have 2 pixels spanning the CFZ) and again converting to a linear equation:

$$CFZ_{camera} = pixel_size . FL . f = pixel_size . f^2 . A \quad (13)$$

Another way of looking at it would be to use the image scale ("/pixel):

$$Image\ Scale = \frac{360 * 60 * 60}{2\pi} . \frac{pixel_size}{FL} = 206,265 . \frac{pixel_size}{FL}$$

Comparison of CFZ calculations

Here is a comparison of CFZ calculations for my equipment:

SkyWatcher Esprit 100ED, FL=550mm, A=100mm, f=5.5

ASI 1600mm Pro: pixel_size=3.8microns

ZWO EAF: 1 tick=5microns

Red=630nm; Green=521nm; Blue=452nm

Sii=672nm; Ha=656nm; Oiii=500nm

CFZ = $4.88.t.\lambda.f^2$		CFZ (Microns)							CFZ (Ticks)						
		Red	Green	Blue	Sii	Ha	Oiii	Camera	Red	Green	Blue	Sii	Ha	Oiii	Camera
t	1	93	77	67	99	97	74	11	19	15	13	20	19	15	2
t	0.33	31	26	22	33	32	25	11	6	5	4	7	6	5	2
CFZ = $4.n.\lambda.f^2$		CFZ (Microns)							CFZ (Ticks)						
		Red	Green	Blue	Sii	Ha	Oiii	Camera	Red	Green	Blue	Sii	Ha	Oiii	Camera
n	0.33	25	21	18	27	26	20	11	5	4	4	5	5	4	2
n	0.1	8	6	5	8	8	6	11	2	1	1	2	2	1	2
CFZ = $0.00225.\sqrt{t}.\theta.f^2.A$		CFZ (Microns)							CFZ (Ticks)						
		$\theta = 4''$	$\theta = 3.5''$	$\theta = 3''$	$\theta = 2.5''$	$\theta = 2''$	$\theta = 1.5''$	Camera	$\theta = 4''$	$\theta = 3''$	$\theta = 2''$	$\theta = 2.5''$	$\theta = 2''$	$\theta = 1.5''$	Camera
t	15	105	92	79	66	53	40	38	21	18	16	13	11	8	8
t	10	86	75	65	54	43	32	31	17	15	13	11	9	6	6
t	5	61	53	46	38	30	23	22	12	11	9	8	6	5	4
CFZ Camera = $\text{pixel_size}.f^2.A$		11 microns			2 Ticks										
Image Scale		1.43 "/pixel													